Int. J. Heat Mass Transfer. Vol. 20, pp. 1045–1050. Pergamon Press 1977. Printed in Great Britain

THERMAL CONVECTION IN A HORIZONTAL POROUS LAYER WITH INTERNAL HEAT SOURCES

MORTEN TVEITEREID
Department of Mechanics, University of Oslo, Oslo, Norway

(*Received* 4 *October* 1976)

Abstract—Steady solutions in the form of hexagons and two-dimensional rolls are obtained for convection in a horizontal porous layer heated from within. The stability of the flows with respect to small disturbances is investigated. It is found that down-hexagons are stable for Rayleigh numbers R up to 8 times the critical value ($8R_c$), while up-hexagons are unstable for all values of R. Moreover, two-dimensional rolls are found to be stable in the range $3R_c < R < 7R_c$. Good agreement with some of the experimental observations of Buretta [1] is found.

NOMENCLATURE

B_{pqh}, defined by (3.5);
C_p, heat capacity at constant pressure;
M, defined by (3.8);
N, Nusselt number;
Q, generated heat per unit time;
R, Rayleigh number;
R_c, critical Rayleigh number;
T, temperature;
T_0, standard temperature;
T_s, defined by (2.5);
V, defined by (3.1);
ΔT, mean temperature difference between the boundaries;
ΔT_0, temperature difference between the boundaries for pure heat conduction;
∇, $= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$;
∇^2, $= \nabla \cdot \nabla$;
∇_1^2, $= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$;
a, wave number;
g, acceleration due to gravity;
h, depth of the layer;
k, permeability;
p, pressure;
$\mathbf{v}$, $= (u, v, w)$, velocity;
t, time;
x, y, z, cartesian coordinates.

Greek symbols

α, coefficient of expansion;
$\boldsymbol{\delta}$, $= \left(\frac{\partial^2}{\partial x \partial z}, \frac{\partial^2}{\partial y \partial z}, -\nabla_1^2\right)$;
θ, temperature;
κ_m, thermal diffusivity;
μ, viscosity;
ρ_0, standard density;
σ, growth rate.

Superscripts

$\tilde{}$, perturbation quantities;
$*$, complex conjugate quantities.

1. INTRODUCTION

THIS paper is concerned with thermal convection in a porous medium. The convective motion is generated by internal heat sources which give a basic temperature gradient dT/dz varying with the vertical coordinate z. Porous convection is of considerable geophysical and technical interest, as it may occur in geothermal areas, through aquifers, oil reservoirs, snow layers, etc. (Combarnous and Bories [2]).

Porous convection when dT/dz being a constant, has been investigated by several authors. Horton and Rogers [3] and later Lapwood [4] determined analytically that above a certain dimensionless temperature gradient, convection can occur. Laboratory experiments have been performed by Schneider [5], Elder [6], Bories and Combarnous [7] and others. Theoretical and numerical analysis of finite amplitude convection have been performed among others by Elder [6], Palm, Weber and Kvernvold [8], Strauss [9] and Kvernvold [10].

In physical problems, however, a constant temperature gradient generally does not occur. Vertical variations of dT/dz may be due to variation in time of the temperature at the boundaries, or due to vertical variations of the thermal diffusivity for the porous medium. In the present analysis, however, the variation of dT/dz is thought of being due to uniformly distributed internal heat sources, which give a simple expression for the basic temperature.

To our knowledge, almost no research has been reported dealing with convection in a porous layer where dT/dz depends on z. Hwang [11] has studied the stability problem of convection in a porous layer with uniform heating from within and from below. He found that the critical Rayleigh number, R_c, decreases, as the effect of internal heating increases. For a model similar to the present model experimental studies have been performed by Buretta [1]. He measured the convective heat transport through the medium for Rayleigh numbers up to about $30R_c$. At a supercritical Rayleigh number, which appeared to depend on layer properties, a discontinuous jump in the convective heat transfer occured. In view of this he postulated that R_c is a bifurcation point beyond which two finite amplitude modes of convection are possible.

In this paper we shall calculate the critical Rayleigh number. Moreover, we shall derive steady solutions by a numerical technique, and examine the stability of these solutions with respect to small disturbances.

2. GOVERNING EQUATIONS

We consider a horizontally infinite layer of porous material saturated with fluid and heated from within by a uniform distribution of heat sources. The layer is bounded by two horizontal and impermeable planes separated by a distance h. The upper plane is taken to be perfect heat conductor and maintained at constant temperature, and the lower plane is taken to be perfect heat insulator.

In the Boussinesq approximation the equations governing the motion of the fluid may be written (Palm and Weber [12])

$$\nabla p+\rho_0[1-\alpha(T-T_0)]g\mathbf{k}+\frac{\mu}{k}\mathbf{v}=0 \qquad (2.1)$$

$$\nabla\cdot\mathbf{v}=0 \qquad (2.2)$$

$$\frac{(C_p)_m}{C_p}\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T=\kappa_m\nabla^2 T+\frac{Q}{C_p}. \qquad (2.3)$$

Here (2.1) is the equation of motion, (2.2) the continuity equation and (2.3) the heat equation. Moreover, p denotes the pressure, $\mathbf{v}=(u, v, w)$ the velocity, T the temperature, ρ_0 a reference density, T_0 a reference temperature, α the coefficient of expansion, g the acceleration due to gravity, $\mathbf{k}$ a unit vector directed upwards, μ the dynamic viscosity, k the permeability, κ the thermal diffusivity, C_p the heat capacity at constant pressure and Q the generated heat in the layer per unit time. The subscript m denotes properties of the fluid–solid mixture. We have chosen a cartesian coordinate system (x, y, z) where the z-axis is directed upwards.

With the lower boundary at $z=0$ the equations (2.1)–(2.3) are subjected to the boundary conditions

$$\begin{aligned} &w=0, \quad \frac{\partial T}{\partial z}=0; \quad z=0\\ &w=0, \quad T=0; \quad z=h. \end{aligned} \qquad (2.4)$$

The temperature scale is chosen such that the constant temperature of the upper boundary is equal to zero.

When Q is small, the heat transfer is in the form of conduction ($\mathbf{v}=0$). Let the static pressure and the conduction temperature then be denoted by p_s and T_s, respectively. From (2.3) and (2.4) we find that

$$T_s=\frac{Q}{2C_p\kappa_m}(h^2-z^2). \qquad (2.5)$$

For larger Q, in the convective regime, we write

$$\mathbf{v}=\mathbf{v}', \quad p=p_s+p', \quad T=T_s+\theta'. \qquad (2.6)$$

The equations may be written in a non-dimensional form by choosing h as a characteristic scale for length, κ_m/h for velocity, $(C_p)_m h^2/C_p\kappa_m$ for time, $\mu\kappa_m/k$ for pressure, $\Delta T_0/R$ for θ' and ΔT_0 for T_s. Here ΔT_0 is the temperature difference between the planes for pure heat conduction and R the Rayleigh number, defined by

$$\Delta T_0=\frac{Qh^2}{2C_p\kappa_m}, \qquad R=\frac{\rho_0 kg\alpha\Delta T_0 h}{\mu\kappa_m}. \qquad (2.7)$$

Omitting the primes the equations (2.1)–(2.4) then take the non-dimensional form

$$\nabla p-\theta\mathbf{k}+\mathbf{v}=0 \qquad (2.8)$$

$$\nabla\cdot\mathbf{v}=0 \qquad (2.9)$$

$$\nabla^2\theta+2Rzw=\frac{\partial\theta}{\partial t}+\mathbf{v}\cdot\nabla\theta \qquad (2.10)$$

with the boundary conditions

$$\begin{aligned} &w=\frac{\partial\theta}{\partial z}=0; \quad z=0\\ &w=\theta=0; \quad z=1. \end{aligned} \qquad (2.11)$$

The linearized version of (2.8)–(2.11) is an eigenvalue problem with $R=R_0$ as the eigenvalue. Introducing the wave number a, defined by

$$\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=-a^2 \qquad (2.12)$$

R_0 becomes a function of a. The minimum value R_c of R_0 for $a=a_c$ defines the value of the Rayleigh number for the onset of convection. $R_0=R_0(a)$ is calculated by developing the solution in a power series of z. By applying 50 terms of this series, we found that

$$R_c=30.933, \qquad a_c=2.448. \qquad (2.13)$$

This result was checked by taking into account 100 terms without obtaining any changes of the given values.

3. STEADY SOLUTIONS

It follows from (2.8) and (2.9) that the velocity is poloidal, giving

$$\mathbf{v}=\boldsymbol{\delta}V=\left(\frac{\partial^2}{\partial x\partial z}, \frac{\partial^2}{\partial y\partial z}, -\nabla_1^2\right)V. \qquad (3.1)$$

Thus, by eliminating the pressure term we obtain from (2.8)–(2.11)

$$\nabla^2 V=-\theta \qquad (3.2)$$

$$\nabla^2\theta-2Rz\nabla_1^2 V=\mathbf{v}\cdot\nabla\theta \qquad (3.3)$$

with the boundary conditions

$$\begin{aligned} &V=\frac{\partial\theta}{\partial z}=0; \quad z=0\\ &V=\theta=0; \quad z=1. \end{aligned} \qquad (3.4)$$

Considering solutions which are Fourier modes in the x,y-coordinates, θ may be written

$$\theta=\sum_{h=1}^{\infty}\sum_{p,q=-\infty}^{\infty}B_{pqh}\,e^{i(pkx+qly)}\cos(h-\tfrac{1}{2})\pi z. \qquad (3.5)$$

Here k and l are the components of the wave number vector in the x- and y-direction, respectively. Moreover, $B_{pqh} = B^*_{-p-qh}$ which ensure that (3.5) is real. The star denotes the complex conjugate. Corresponding to (3.5), V may be written

$$V = \sum B_{pqh} e^{i(pkx+qly)} F_h(\kappa, z) \tag{3.6}$$

where $\kappa^2 = (pk)^2 + (ql)^2$. $F_h(\kappa, z)$ is found from (3.2) and (3.4) (see Appendix).

Introducing (3.5) and (3.6) into (3.3), multiplying this by $\exp\{-i(rkx+sly)\}\cos(g-\frac{1}{2})\pi z$ and averaging over the layer, we obtain a system of equations which determine the unknown coefficients B_{rsg}:

$$\tfrac{1}{2}\{(g-\tfrac{1}{2})^2\pi^2+\nu^2\}B_{rsg} - 2R\nu^2 \sum_h a(h,\nu,g)B_{rsh} - \sum_{h,f}\sum_{\substack{p+t=r\\ q+u=s}} \{(ptk^2+qul^2)b(h,\kappa,f,g) + \kappa^2 c(h,\kappa,f,g)\}B_{pqh}B_{tuf} = 0 \tag{3.7}$$

where $\nu^2 = (rk)^2 + (sl)^2$. The coefficients a, b and c are given in the Appendix.

The system of equations (3.7) has many different types of possible solutions. We shall, however, limit our analysis to the possibility of flow patterns consisting of hexagons or two-dimensional rolls. Probably, as in the case of free convection in a horizontal fluid layer, only hexagons and two-dimensional rolls can be stable flows for moderate Rayleigh numbers (Segel [13], Palm [14], Tveitereid and Palm [16]). To obtain this two types of flow as solutions of (3.7), we may require that all B_{rsg} are real, $B_{rsg} = B_{r-sg}$, $r+s$ equal to an even number, and that $k^2 = 3l^2F = 3a^2/4$. Moreover, we truncate the infinite system by neglecting all modes for which

$$g^2 + 3r^2/4 + s^2/4 > M^2 + 1. \tag{3.8}$$

Here M is an integer. In order to specify the values to be used for M, we introduce the Nusselt number N_M defined by

$$N_M = \Delta T_0/\Delta T = 1\bigg/\left(1+\sum_{g=1}^{M} B_{00g}/R\right) \tag{3.9}$$

where ΔT is the mean temperature difference between the planes and ΔT_0 the temperature difference in the case of pure heat conduction. If N_M differs from N_{M-1} by less than 1%, the solution is accepted to be sufficiently accurate.

By using a Newton–Raphson method to solve (3.7), we find that two-dimensional rolls and both down-hexagons (i.e. descending flow in the centre of the cells) and up-hexagons are steady state configurations of our problem.

4. STABILITY ANALYSIS

Let $\tilde{\theta}$ and $\tilde{\mathbf{v}} = \boldsymbol{\delta}\tilde{V}$ denote a small variation of θ and $\mathbf{v}$, respectively. Furthermore, we assume an exponential time dependence such that

$$\frac{\partial}{\partial t}\tilde{\theta} = \sigma\tilde{\theta} \tag{4.1}$$

where σ is the growth rate. By eliminating the pressure term, replacing $\mathbf{v}$ with $\mathbf{v}+\tilde{\mathbf{v}}$ and θ with $\theta+\tilde{\theta}$ in (2.8)–(2.11), the equations governing the perturbations are

$$\nabla^2\tilde{V} = -\tilde{\theta} \tag{4.2}$$

$$\nabla^2\tilde{\theta} - 2Rz\nabla_1^2\tilde{V} = \sigma\tilde{\theta} + \tilde{\mathbf{v}}\cdot\nabla\theta + \mathbf{v}\cdot\nabla\tilde{\theta} \tag{4.3}$$

with the boundary conditions

$$\tilde{V} = \frac{\partial\tilde{\theta}}{\partial z} = 0; \quad z = 0$$
$$\tilde{V} = \tilde{\theta} = 0; \quad z = 1. \tag{4.4}$$

Assuming periodical solutions in x and y, θ may be written

$$\tilde{\theta} = e^{i(\varepsilon kx+\delta ly)} \sum \tilde{B}_{pqh} e^{i(pkx+qly)} \cos(h-\tfrac{1}{2})\pi z \tag{4.5}$$

where ε and δ are free parameters.

To obtain a complete stability analysis of the hexagonal flow δ and ε are varied from zero to one and from zero to $\delta/3$, respectively. For two-dimensional rolls with axis parallel the x-axis δ and ε are varied from zero to one and from zero to infinity, respectively.

From (4.2) and (4.5) we obtain

$$\tilde{V} = e^{i(\varepsilon kx+\delta ly)} \sum \tilde{B}_{pqh} e^{i(pkx+qly)} F_h(\tilde{\kappa}, z) \tag{4.6}$$

where $\tilde{\kappa}^2 = (p+\varepsilon)^2k^2 + (q+\delta)^2l^2$. We introduce (4.5) and (4.6) into (4.3), multiply with

$$\exp\{-i(\varepsilon kx+\delta ly)\}\exp\{-i(rkx+sly)\}\cos(g-\tfrac{1}{2})\pi z$$

and average over the layer. Then, an infinite set of linear and homogeneous equations determining $\tilde{B}_{rsg}$, follows. As in the previous section, we take into account only those equations for which $g^2 + \frac{3}{4}r^2 + \frac{1}{4}s^2 \leqslant M^2 + 1$. The stability problem is thus reduced to an eigenvalue problem with σ as the eigenvalue. If for given R and a at least one of the eigenvalues has positive real part, the examined flow is unstable.

5. RESULTS AND DISCUSSION

Figure 1 shows the results of the stability calculations. We find that down-hexagons and two-dimensional rolls are stable in a region of the (a, R)-plane, while up-hexagons are unstable for all values of a and R.

Hexagons

The down-hexagons are stable in a rather small part of the wave number range from R_c up to $8R_c$. The stable region is tilted to the right, such that the wavelength of the cells at $8R_c$ is almost halved compared with the wavelength at R_c. Most of the curve which encloses the stable region (the neutral curve), is defined by non-oscillatory disturbances (i.e. $\sigma = 0$). From $4R_c$ up to $8R_c$, however, the left branch of the neutral curve is defined by oscillatory distrubances (i.e. the imaginary part of σ is different from zero). Moreover, also a subcritical region is found. This is, however, very small ($30.91 < R < R_c = 30.93$) and is of no practical interest.

The Nusselt number is illustrated in Fig. 2 for $M = 5$ and 6. We observe that N_6 differs from N_5 by less than 1%. This small difference, together with almost the same stable region for $M = 5$ and 6 (see Fig. 1),

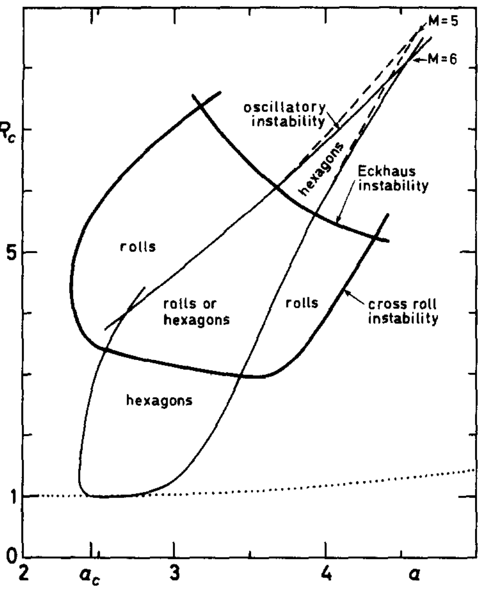

FIG. 1. The stable regions. ——, the neutral curve for down-hexagons. Stable inside, unstable outside. ▬▬, the neutral curve for two-dimensional rolls. Stable inside, unstable outside. ·····, the marginal stable curve.

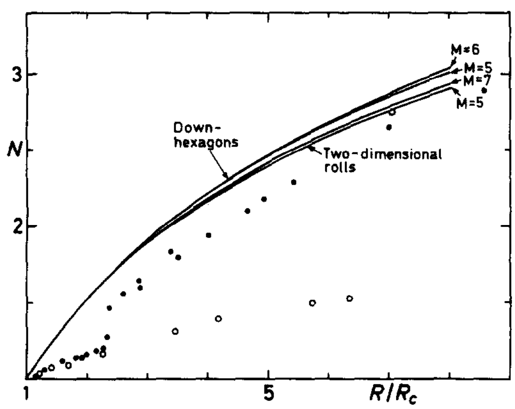

FIG. 2. The Nusselt number as a function of the Rayleigh number. ○, ●, values of the Nusselt number from Buretta [1].

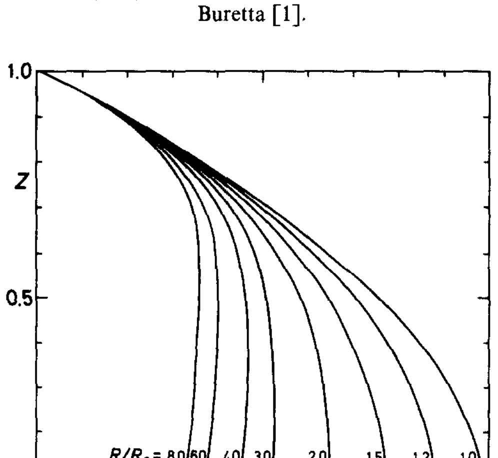

FIG. 3. The horizontally averaged temperature for the hexagonal flow.

indicates that $M = 6$ defines an acceptable truncation of the equations.

The horizontally averaged temperature field, given by

$$\bar{T} = T_s + \frac{1}{R} \sum_{g=1} B_{00g} \cos(g - \tfrac{1}{2})\pi z \qquad (5.1)$$

is shown in Fig. 3. As R is increased above R_c, we observe that the interior and the lower part of the layer become nearly isothermal, while a "thermal-boundary layer" is formed in the upper part of the layer.

Rolls

Two-dimensional rolls are stable in a broad wave number range from $3R_c$ up to $7R_c$. There is two types of disturbances which define the neutral curve. The right and the left branch are defined by cross roll disturbances, while the top branch is defined by Eckhaus disturbances (for a review of these types of disturbances: see Busse [15]). In the present case both the cross roll instability and the Eckhaus instability are non-oscillatory. To our knowledge the Eckhaus instability has never been observed in experiments. This is because the Eckhaus instability usually becomes important only for small supercritical Rayleigh numbers. The present result shows, however, that it should be possible to study the Eckhaus mechanism also in experiments.

The Nusselt number for steady two-dimensional rolls are given in Fig. 2 for $M = 5$ and 7. We notice that N_7 differs from N_5 by less than 1%. Also the stable region is calculated for $M = 5$ and 7. We find, however, the same neutral curve for these two values of M. Moreover, we observe the very small difference between the Nusselt numbers for rolls and hexagons. This fact supports the frequently used assumption that the convective heat transfer in a convection layer is nearly independent of the planform of the motion.

By comparing our results in the present work with the results in [16, 17] we find agreement as to the sign of the circulation in the hexagonal cells. In these papers we found that down-hexagons are stable and up-hexagons unstable when the second derivative of the basic temperature is less than zero (as in the present case) and vice versa. The sign of the circulation is also in accordance with the observations in [18, 19].

For Rayleigh numbers from $3R_c$ up to $7R_c$ we found both stable rolls and stable hexagons. This bifurcation phenomenon must be due to properties of the porous medium. Since, in [16], where we studied convection in a fluid layer subjected to the same thermal conditions as the present ones, only down-hexagons were found to be stable.

Also different from the results in [16] is our finding of an upper limit of stable motion. However, the occurrence of unstable convection above $8R_c$ is probably caused by thermal instability of the thermal boundary layer. Let δ and R_δ denote the dimensionless thickness and the Rayleigh number of the boundary layer, respectively. Then

$$R_\delta = R\bar{T}(z = 1 - \delta)\delta. \qquad (5.2)$$

If $\bar{T}(z = 1-\delta)\delta > 1/8$ at $R = 8R_c$, R_δ becomes larger than R_c. From Fig. 3 we observe that this condition may be fulfilled. In a fluid layer, however, where R_δ is proportional to δ^3, R_δ does not become larger than R_c for moderate values of R.

Finally, also shown in Fig. 2 are the experimental values of N found by Buretta [1]. At a supercritical Rayleigh number R_D, depending on the diameter of the beads, he observed a remarkable discontinuity in the convective heat transfer. For R higher than R_D our numerical values of N agree very well with the experimental values. For R less than R_D, however, the agreement is rather bad. This discrepancy, we believe, is caused by experimental difficulties.

6. SUMMARY

In this paper we have studied finite amplitude convection in a porous medium heated from within. Three different steady flows are analysed: down-hexagons, up-hexagons and two-dimensional rolls. By examining the stability of the flows with respect to small disturbances, the down-hexagons and the rolls are found to be stable planforms. The results of the stability analysis are shown in Fig. 1. Worth mentioning is also the occurrence of oscillatory instability of the down-hexagons. From $4R_c$ up to $8R_c$ the left branch of the neutral curve was defined by disturbances having a complex growth rate.

Moreover, in Fig. 2 we have compared the convective heat transfer with experimental values obtained by Buretta [1]. Some of the experimental values show good agreement with our values.

REFERENCES

1. R. Buretta, Thermal convection in a fluid filled porous layer with uniform internal heat sources, Ph.D. Thesis, Univ. of Minnesota (1972).
2. M. A. Combarnous and S. A. Bories, Hydrothermal convection in saturated porous media, *Adv. Hydrosci.* **10**, 232 (1975).
3. C. W. Horton and F. T. Rogers, Convection currents in a porous medium, *J. Appl. Phys.* **16**, 367 (1945).
4. E. R. Lapwood, Convection of fluid in a porous medium, *Proc. Camb. Phil. Soc.* **44**, 508 (1948).
5. K. J. Schneider, Investigation of the influence of free thermal convection on heat transfer through granular materials, 11th Int. Cong. of Refrigeration, Munich, paper 11-4 (1963).
6. J. W. Elder, Steady free convection in a porous medium heated from below, *J. Fluid Mech.* **27**, 29 (1967).
7. S. A. Bories and M. A. Combarnous, Natural convection in a sloping porous layer, *J. Fluid Mech.* **57**, 63 (1973).
8. E. Palm, J. E. Weber and O. Kvernvold, On steady convection in a porous medium, *J. Fluid Mech.* **54**, 153 (1972).
9. J. M. Strauss, Large amplitude convection in porous media, *J. Fluid Mech.* **64**, 51 (1974).
10. O. Kvernvold, Non-linear convection in a porous medium, Appl. Math., Univ. of Oslo, Preprint Ser. No. 1 (1975).
11. I.-T. Hwang, Finite amplitude thermal convection in porous media with heat sources and variable viscosity, Ph.D. Thesis, Univ. of Minnesota (1971).
12. E. Palm and J. E. Weber, Derivation of the equations for thermal convection in a porous material, Appl. Math., Univ. of Oslo, Preprint Ser. No. 4 (1971).
13. L. A. Segel, The non-linear interaction of a finite number of disturbances to a layer of fluid heated from below, *J. Fluid Mech.* **21**, 359 (1965).
14. E. Palm, Non-linear thermal convection, *Ann. Rev. Fluid Mech.* **7**, 39 (1975).
15. F. H. Busse, Stability regions of cellular fluid flow, in *Proc. IUTAM-Symp., Herrenalb* 1969: *Instability of Continuous Systems*. Springer, Berlin (1969).
16. M. Tveitereid and E. Palm, Convection due to internal heat sources, *J. Fluid Mech.* **76**, 481 (1976).
17. M. Tveitereid, Thermal convection in a horizontal fluid layer with internal heat sources, Appl. Math., Univ. of Oslo, Preprint Ser. No. 2 (1976).
18. D. J. Tritton and M. N. Zarraga, Convection in horizontal layers with internal heat generation, Experiments, *J. Fluid Mech.* **30**, 29 (1967).
19. E. W. Schwiderski and H. J. A. Schwab, Convection experiments with electrolytically heated fluid layers, *J. Fluid Mech.* **48**, 703 (1971).

APPENDIX

Definition of the Function $F_h(\kappa, z)$

From (3.2)–(3.6) we obtain

$$\left(\frac{d^2}{dz^2} - \kappa^2\right)F_h(\kappa, z) = -\cos(h-\tfrac{1}{2})\pi z \qquad (A.1)$$

with the boundary conditions

$$F_h = 0, \quad z = 0,1. \qquad (A.2)$$

We find

$$F_h(\kappa, z) = A_h(\kappa)\cos(h-\tfrac{1}{2})\pi z + C_h^{(1)}(\kappa)\,e^{-\kappa z} + C_h^{(2)}(\kappa)\,e^{\kappa z} \qquad (A.3)$$

where

$$\begin{aligned} A_h(\kappa) &= 1/[(h-\tfrac{1}{2})^2\pi^2 + \kappa^2] \\ C_h^{(1)}(\kappa) &= -A_h(\kappa)\,e^{\kappa}/(e^{\kappa} - e^{-\kappa}) \\ C_h^{(2)}(\kappa) &= A_h(\kappa)\,e^{-\kappa}/(e^{\kappa} - e^{-\kappa}). \end{aligned} \qquad (A.4)$$

Definition of the Coefficients a, b *and* c

$$\begin{aligned} a(h, \nu, g) &= \int_0^1 zF_h(\nu, z)\cos(g-\tfrac{1}{2})\pi z\,dz \\ b(h, \kappa, f, g) &= \int_0^1 F_h'(\kappa, z)\cos(f-\tfrac{1}{2})\pi z\cos(g-\tfrac{1}{2})\pi z\,dz \\ c(h, \kappa, f, g) &= (f-\tfrac{1}{2})\pi\int_0^1 F_h(\kappa, z)\sin(f-\tfrac{1}{2})\pi z\cos(g-\tfrac{1}{2})\pi z\,dz. \end{aligned} \qquad (A.5)$$

CONVECTION THERMIQUE DANS UNE COUCHE POREUSE ET HORIZONTALE AVEC DES SOURCES DE CHALEUR INTERNES

Résumé—On obtient des solutions permanentes sous la forme d'hexagones et de rouleaux bidimensionnels, pour la convection dans une couche poreuse horizontale et chauffée intérieurement. On étudie la stabilité des écoulements vis à vis de petites perturbations. On trouve que les hexagones descendants sont stables pour des nombres de Rayleigh R allant jusqu'à huit fois la valeur critique ($8R_c$), tandis que les hexagones ascendants sont instables pour toutes les valeurs de R. Les rouleaux bidimensionnels sont stables dans le domaine $3R_c < R < 7R_c$. On trouve un bon accord avec quelques observations expérimentales de Buretta [1].

DIE THERMISCHE KONVEKTION IN EINER HORIZONTALEN PORÖSEN SCHICHT MIT INNEREN WÄRMEQUELLEN

Zusammenfassung—Für die Konvektion in einer horizontalen porösen Schicht mit inneren Wärmequellen werden stationäre Lösungen in Form hexagonaler und zweidimensionaler Rollzellen erhalten. Die Stabilität dieser Strömungen wird in bezug auf kleine Störungen untersucht. Abwärtsgerichtete Hexagonalzellen erweisen sich bis zur 8-fachen kritischen Rayleigh–Zahl als stabil; aufwärtsgerichtete Hexagonalzellen sind für alle Rayleigh–Zahlen instabil. Zweidimensionale Rollzellen sind für Rayleigh–Zahlen zwischen der 3-fachen und 7-fachen kritischen Rayleigh–Zahl stabil. Es liegt eine gute Übereinstimmung mit einigen der Beobachtungen von Buretta [1] vor.

КОНВЕКТИВНЫЙ ТЕПЛООБМЕН В ГОРИЗОНТАЛЬНОМ ПОРИСТОМ СЛОЕ С ВНУТРЕННИМИ ИСТОЧНИКАМИ ТЕПЛА

Аннотация — Стационарное решение конвекции в горизонтальном пористом нагреваемом изнутри слое дает структуру конвективного движения в виде шестиугольников и двумерных валов. Исследуется устойчивость течения по отношению к небольшим возмущениям. Найдено, что шестиугольники в нижней части слоя проявляют устойчивость при значениях числа Релея, R, в восемь раз превышающих критическое значение, $8R_c$, в то время как шестиугольники в верхней части неустойчивы при любых значениях R. Кроме того найдено, что двухмерные валы проявляют устойчивость в диапазоне $3R_c < R < 7R_c$. Получено хорошее соответствие с экспериментальными данными работы Баретта [1].